Counting Statistics

Sources of Error

- Systematic errors
  - Consistently get the same error
- Random errors
  - Radiation emission and detection are random processes
- Blunder
  - operator error

Measures of Central Tendency

- Mean
  - Average value
- Median
  - Middlemost measurement (or value)
  - Less affected by outliers

Example: 8, 14, 5, 9, 12
Mean = 9.6
Median = 9

Measures of Variability

- Variance
  - Measure of variability:

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_N - \bar{x})^2}{N - 1}$$

- Standard deviation
  - Square root of variance

$$\sigma = \sqrt{\sigma^2}$$

Statistical Models for random trials

- Binomial Distribution
- Poisson Distribution
  - Simplification of binomial distribution with certain constraints
- Gaussian or Normal Distribution
  - Further simplification if average number of successes is large (e.g., >20)

Binomial process

- Trial can have only two outcomes

Trial	Definition of a success	Probability of a success
Toss of a coin	"Heads"	1/2
Toss of a die	"A four"	1/6
Observation of a radioactive nucleus for a time "*t*"	It decays	$1 - e^{-\lambda t}$
Observation of a detector of efficiency E placed near a radioactive nucleus for a time "*t*"	A count	$E(1 - e^{-\lambda t})$

Source: Adapted from Knoll, GF. *Radiation detection and measurement,* 3rd ed. New York: John Wiley, 2000.

Binomial probability density function (PDF)

$$P(x) = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

- N is total number of trials
- *p* is probability of success
- *x* is number of successes

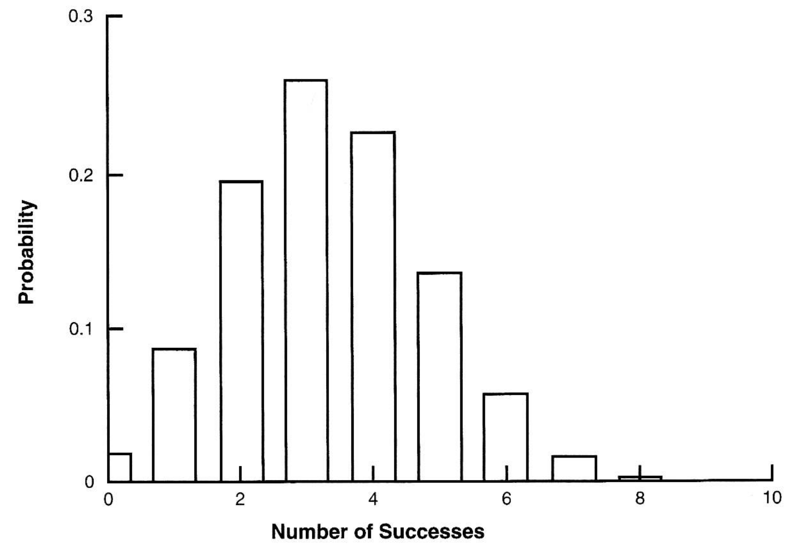


FIGURE 20-28. Binomial probability distribution function when the probability of a success in a single trial (*p*) is 1/3 and the number of trials (*N*) is 10.

Binomial probability density function mean and variance

$$\bar{x} = pN \qquad \text{and} \qquad \sigma = \sqrt{pN(1-p)}$$

- N is total number of trials
- *p* is probability of success
- $\bar{x}$ is mean, σ is standard deviation

If *p* is very small and a constant then:

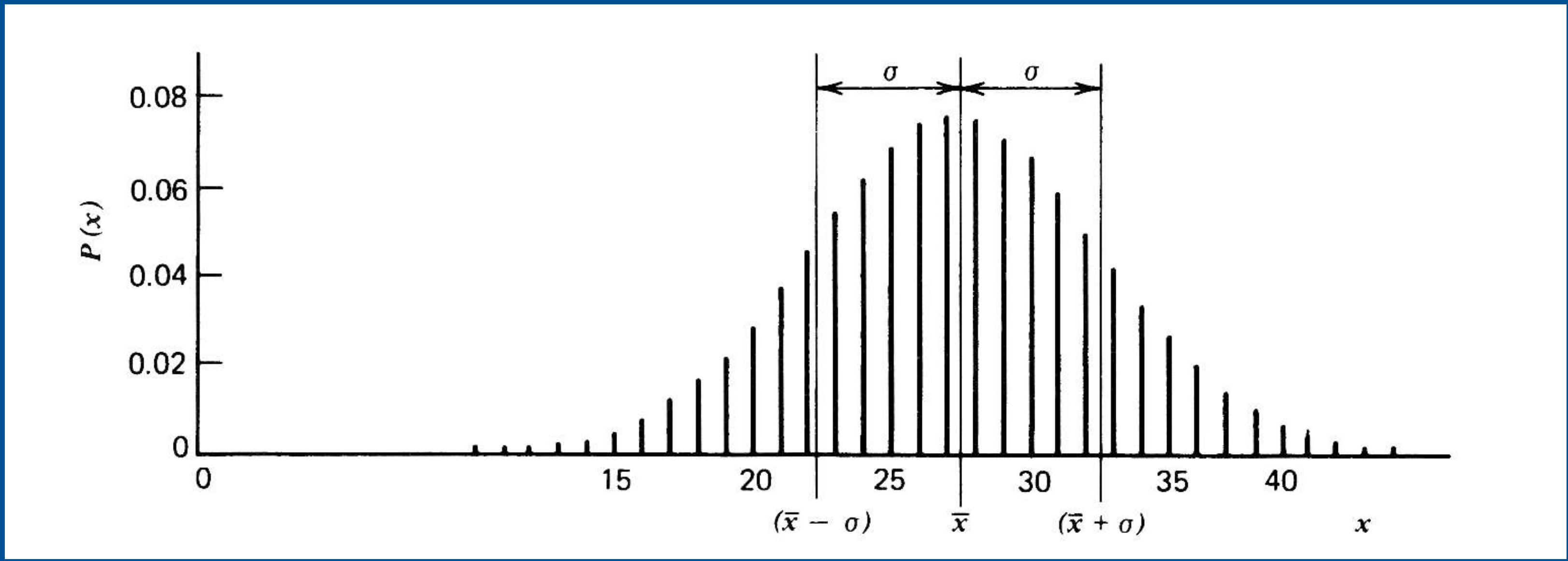
$$\sigma = \sqrt{pN(1-p)} \approx \sqrt{pN} = \sqrt{\bar{x}}$$

Same as Poisson random process.

Poisson PDF

- Radioactive decay and detection are Poisson random processes
  - Observation time is short compared to the half-life of the source
    - probability of radioactive decays (i.e., p)remains constant
    - probability of a given nucleus undergoing decay is small
- Variance
  - Variance = mean = pN = $\bar{x}$
- Standard deviation
  - Standard deviation = $\sqrt{\text{variance}} = \sqrt{pN} = \sqrt{\bar{x}}$
- Can estimate standard deviation from a single measurement

Confidence Intervals



Interval about measurement	Probability that mean is within interval (%)
$\pm 0.674\sigma$	50.0
$\pm 1.0\sigma$	68.3
$\pm 1.64\sigma$	90.0
$\pm 1.96\sigma$	95.0
$\pm 2.58\sigma$	99.0
$\pm 3.0\sigma$	99.7

From: Radiation Detection and Measurement (Knoll, GF)

Raphex Question

D70. How many counts must be collected in an instrument with zero background to obtain an error limit of 1% with a confidence interval of 95%?

A. 1000
B. 3162
C. 10,000
D. 40,000
E. 100,000

Raphex Answer

D70. How many counts must be collected in an instrument with zero background to obtain an error limit of 1% with a confidence interval of 95%?

D. A 95% confidence interval means the counts must fall within two standard deviations (SD) of the mean (N). Error limit = 1% = 2 SD/N, but SD = $N^{1/2}$. Thus $0.01 = 2(N^{1/2})/N = 2/N^{1/2}$. Where $[0.01]^2 = 4/N$ and N = 40,000.

Propagation of Error

Description	Operation	Standard Deviation
Multiplication of a number with random error by a number without random error	cx	$c\sigma$
Division of a number with random error by a number without random error	x/c	σ/c
Addition of two numbers containing random errors	$x_1 + x_2$	$\sqrt{\sigma^2_1 + \sigma^2_2}$
Subtraction of two numbers containing random errors	$x_1 - x_2$	$\sqrt{\sigma^2_1 + \sigma^2_2}$

Raphex question

G74. A radioactive sample is counted for 1 minute and produces 900 counts. The background is counted for 10 minutes and produces 100 counts. The net count rate and net standard deviation are about ____ and ____ counts.

A. 800, 28
B. 800, 30
C. 890, 28
D. 890, 30
E. 899, 30

Raphex answer

G74. A radioactive sample is counted for 1 minute and produces 900 counts. The background is counted for 10 minutes and produces 100 counts. The net count rate and net standard deviation are about ____ and ____ counts/min.

D. The net count rate is:

$$[(N_s/t_s) - (N_b/t_b)] = [(900/1) - (100/10)] = 890.$$

The net standard deviation, σ is:

$$[(N_s/t^2_s) - (N_b/t^2_b)]^{1/2} = [(900) + (1)] = 30.$$